

FourierMotzkinGUI

User Guide

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1 Introduction

FourierMotzkinGUI is a piece of software (M-code) which provides a basic Graphical User Interface for Fourier-Motzkin Elimination (FME) [1] with particular focus on applications to information theory. This algorithm is useful to simplify achievable regions by eliminating meaningless quantities (if possible) but its application by hands can take hours.

The tool is able to work with symbolic quantities, but does not perform any symbolic simplifications. For instance, it can manipulate (formal) quantities $I(U; A)$ and $I(V; A|U)$ but will not write $I(U; A) + I(V; A|U)$ as $I(UV; A)$. Final results will thus contain some redundant inequalities which can be easily eliminated by standard manipulation.

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3 Basic Use

The interface provides a step-by-step procedure as described below.

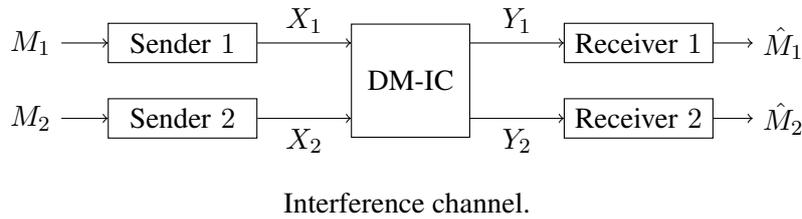
Screenshots are given for the Han-Kobayashi inner bound [2] which provides achievable rates for a 2-user pair discrete memoryless interference channel $(X_1, X_2) \mapsto (Y_1, Y_2)$. In this setup, senders 1 and 2 wish to send independent messages M_1 and M_2 (resp.) with respective rates R_1 and R_2 to receivers 1 and 2 (resp.), as depicted below.

In the coding scheme of [2], message M_j ($j \in \{1, 2\}$) is divided in “public” and “private” parts with respective rates R_{0j} and R_{jj} . Sufficient conditions for the achievability of a tuple $(R_{01}, R_{02}, R_{11}, R_{22})$ are given by the following system of inequalities [2, 3]:

$$\begin{aligned} R_{11} &< I(X_1; Y_1 | U_1, U_2, Q) \\ R_{11} + R_{01} &< I(X_1; Y_1 | U_2, Q) \\ R_{11} + R_{02} &< I(X_1, U_2; Y_1 | U_1, Q) \\ R_{11} + R_{01} + R_{02} &< I(X_1, U_2; Y_1 | Q) \\ R_{22} &< I(X_2; Y_2 | U_1, U_2, Q) \\ R_{22} + R_{02} &< I(X_2; Y_2 | U_1, Q) \\ R_{22} + R_{01} &< I(X_2, U_1; Y_2 | U_2, Q) \\ R_{22} + R_{02} + R_{01} &< I(X_2, U_1; Y_2 | Q) \end{aligned}$$

for some auxiliary variables Q, U_1 and U_2 s.t. $p(qu_1u_2x_1x_2) = p(q)p(u_1x_1|q)p(u_2x_2|q)$.

We would like to eliminate the auxiliary rates and obtain a new system that only involves $R_1 = R_{01} + R_{11}$ and $R_2 = R_{02} + R_{22}$. Fourier-Motzkin elimination is appropriate here (see also its application “by hands” to the same system in [3, Appendix D]).



Step 0. START

At start, it is possible to LOAD AN EXISTING SESSION (see Section 4.2 below), or CREATE A NEW ONE. In the next paragraphs, we present the whole procedure i.e., create a new session.

Step 1. Enter variables

Enter the name of all involved variables in the editable field. Press OK between each one. The LIST OF VARIABLES is automatically updated. Variables are understood as real numbers which describe some regions of \mathbb{R}^d e.g., rates. Some of them are meaningful, others will be eliminated using FME.

You can also EDIT or DELETE a variable already in the list, or CLEAR the whole list, using the appropriate button.

Press ENTER PARAMETERS when finished.

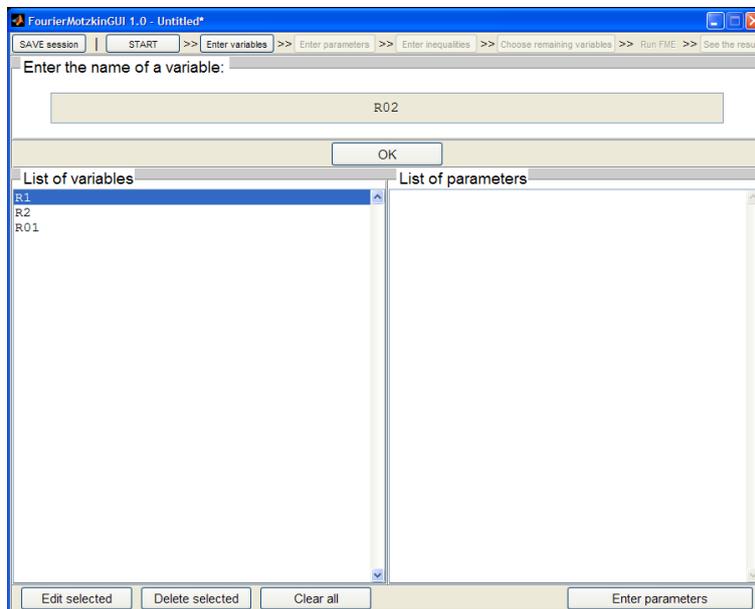


Figure 1: Enter variables.

Step 2. Enter parameters

Enter the name/expression of all involved parameters in the editable field. Press OK between each one. The LIST OF PARAMETERS is automatically updated. Parameters are understood as *non-negative* real numbers e.g., mutual information, entropy...

You can also EDIT or DELETE a parameter already in the list, or CLEAR the whole list, using the appropriate button.

Press ENTER INEQUALITIES when finished.

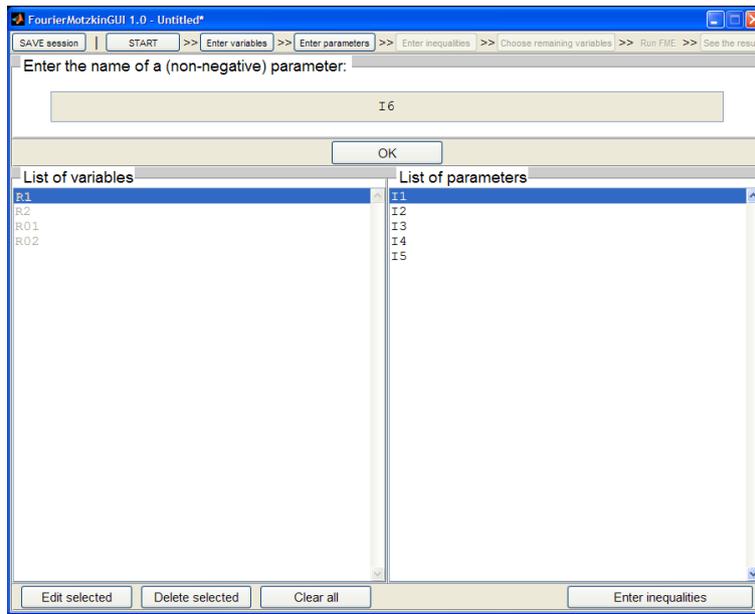


Figure 2: Enter parameters.

The reasons why the parameters are not included in the variables are the following:

- Parameters can not be simplified by FME (they are *constants*),
- We usually write inequalities (some variables) \leq (some parameters),
- Knowing that they are non-negative allows more powerful reduction after application of FME.

Step 3. Enter inequalities

Enter the coefficients (real numbers) of variables/parameters for each inequality of the system in the editable fields. Press OK between each inequality. The LIST OF INEQUALITIES is automatically updated.

You can also EDIT or DELETE an inequality already in the list, or CLEAR the whole list, using the appropriate button.

Press CHOOSE REMAINING VARIABLES when finished.

Step 4. Choose remaining variables

Select the (meaningful) variables that FME should not eliminate i.e., those which are needed to describe the region.

Press RUN FOURIER-MOTZKIN when finished.

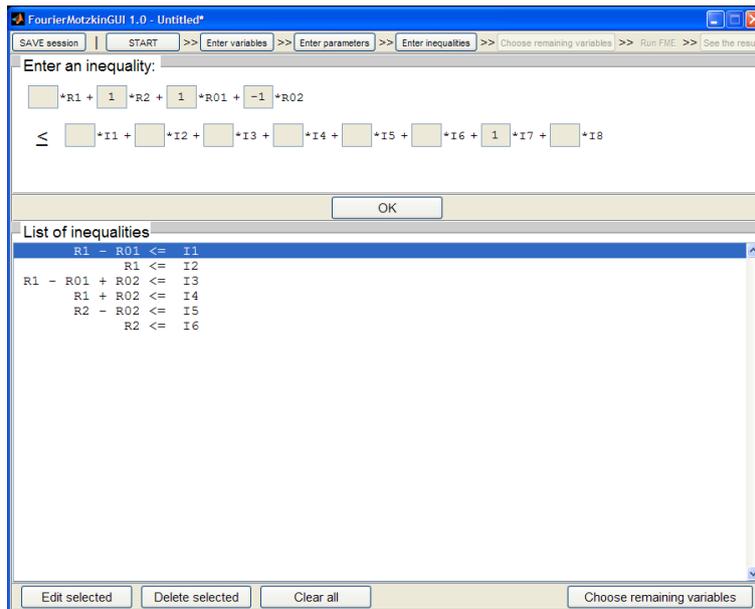


Figure 3: Enter inequalities.

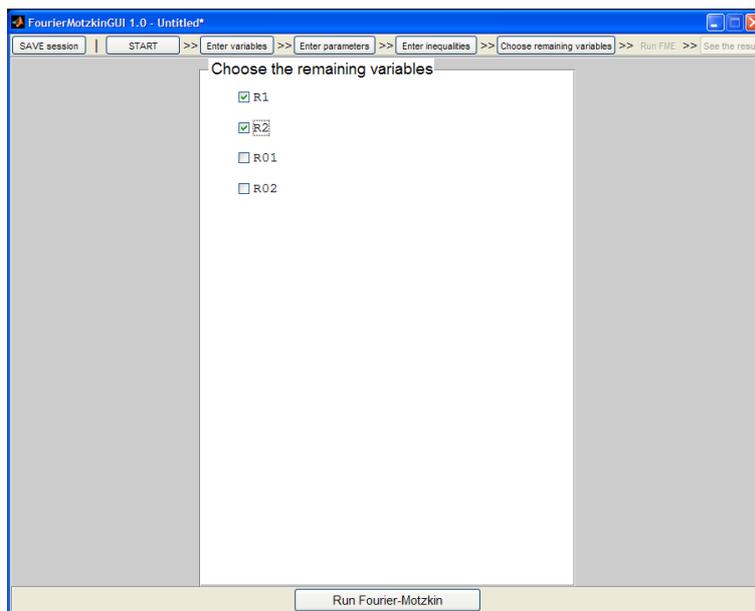


Figure 4: Choose remaining variables.

Step 5. Fourier-Motzkin Elimination is running...

While FME is running, some messages are displayed e.g., the variable currently being eliminated, or the total number of remaining inequalities.

!Warning! The elimination may take a long time, depending on the complexity of the system.

Press CANCEL if you want to interrupt the elimination. Press RESTART FME to restart it (from the beginning).

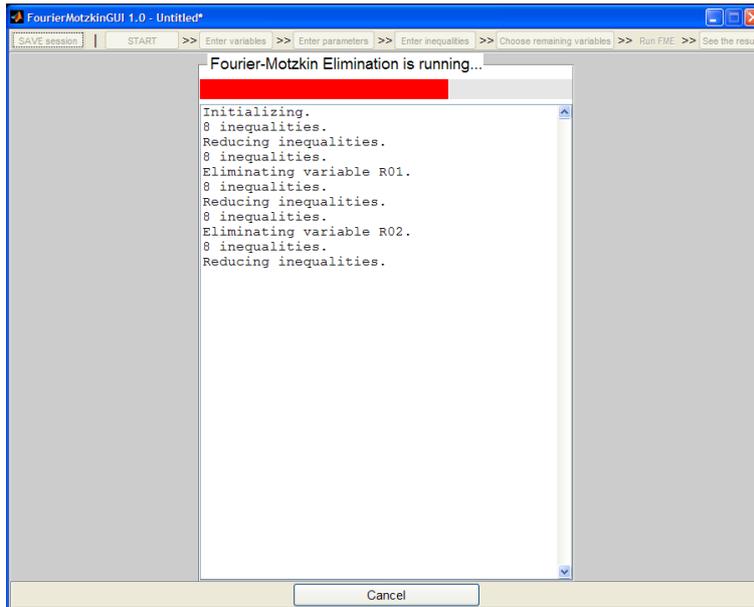


Figure 5: Fourier-Motzkin Elimination is running.

Step 6. See the result

When FME finishes, the initial and reduced systems are displayed. Press EXPORT AS TEXT to save them in a text file.

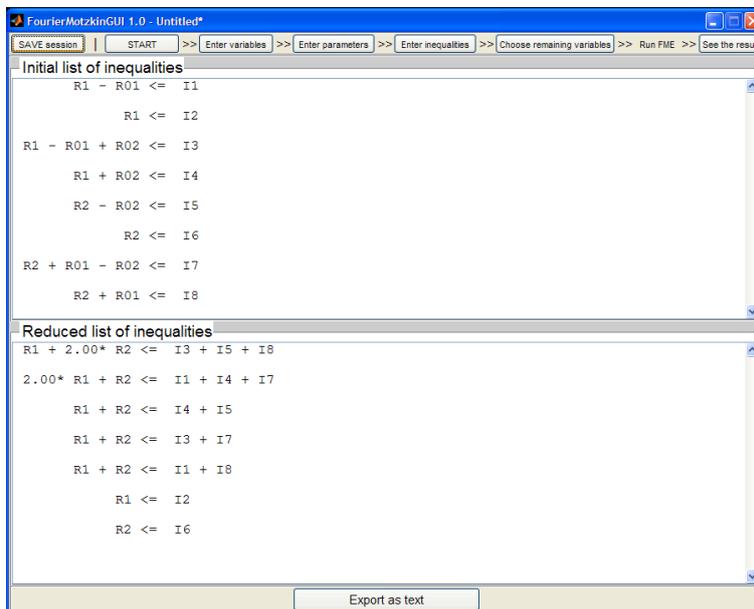


Figure 6: See the result.

The above figure directly gives the expected final system of inequalities [2, 3]:

$$\begin{aligned}
 R_1 + 2R_2 &< I(X_1U_2; Y_1|U_1Q) + I(X_2; Y_2|U_1U_2Q) + I(X_2U_1; Y_2|Q) \\
 2R_1 + R_2 &< I(X_1; Y_1|U_1U_2Q) + I(X_1U_2; Y_1|Q) + I(X_2U_1; Y_2|U_2, Q) \\
 R_1 + R_2 &< I(X_1U_2; Y_1|Q) + I(X_2; Y_2|U_1U_2Q) \\
 R_1 + R_2 &< I(X_1U_2; Y_1|U_1Q) + I(X_2U_1; Y_2|U_2Q) \\
 R_1 + R_2 &< I(X_1; Y_1|U_1U_2Q) + I(X_2U_1; Y_2|Q) \\
 R_1 &< I(X_1; Y_1|U_2Q) \\
 R_1 &< I(X_2; Y_2|U_1Q)
 \end{aligned}$$

4 Others Functionnalities

4.1 Direct Access Buttons

While moving forward, buttons indicating completed steps are turned on in the top menu. They enable to move back to any previous step and make changes if needed.

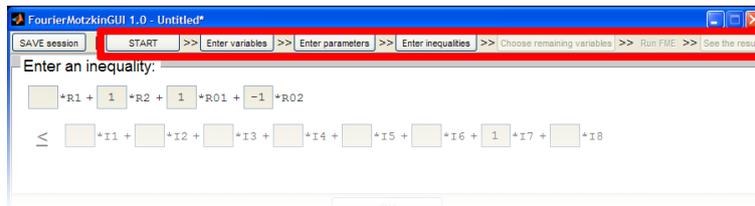


Figure 7: Direct access buttons.

4.2 Save/Load Sessions

Press SAVE SESSION at any time to save the current session i.e., variables, parameters, inequalities,... (depending on the completed steps), in a MAT file.

At start, press LOAD AN EXISTING SESSION, then pick the corresponding MAT file. The program will resume where it was when the session was saved.

References

- [1] Wikipedia.org, “Fourier-Motzkin elimination,” http://en.wikipedia.org/wiki/Fourier-Motzkin_elimination, 7 Oct. 2010.
- [2] T.S. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, 1981.
- [3] A. El Gamal and Y.-H. Kim, *Lecture Notes on Network Information Theory*, 2010.

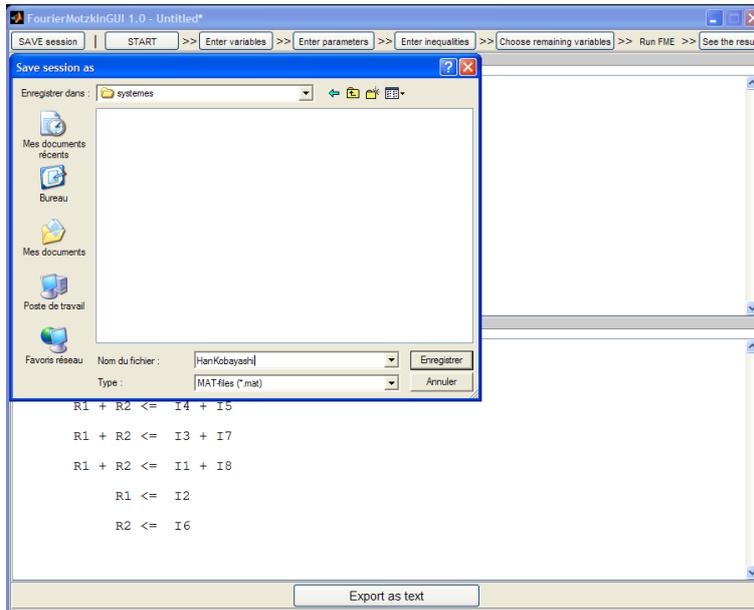


Figure 8: Save a session.

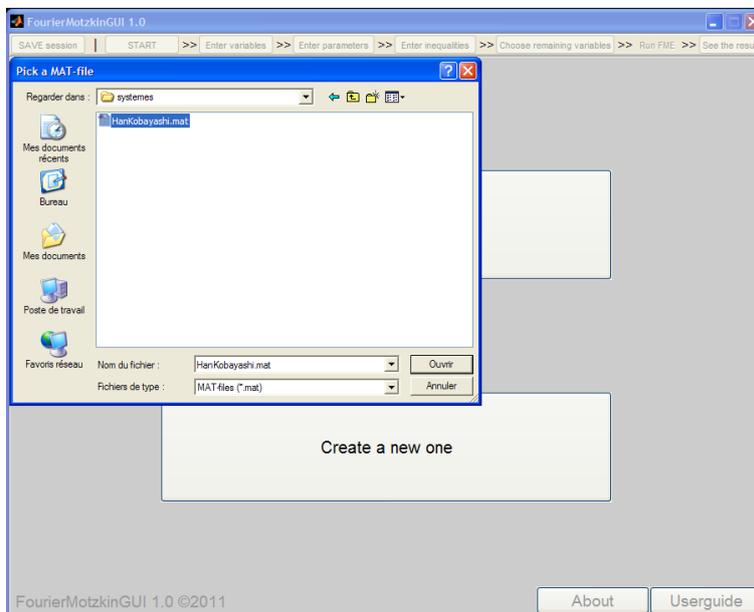


Figure 9: Load a session.

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